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ORIGINAL ARTICLE

Changes in risk and entrepreneurship

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Abstract
In this paper, we extend the existing literature on entrepreneurship by analyzing the effects of changes in risk on two decisions made by the entrepreneur: first, the decision to transit from paid and risk-free employment to risky entrepreneurship, and second, the decision regarding the size or scale of the venture for transitioned entrepreneurs. We provide the conditions that guarantee expected comparative static results under first- and second-order stochastic dominance shifts. We then apply our results to the case of hyperbolic absolute risk aversion preferences, which is a specific functional form commonly used in the economics of risk literature. Interesting results arise from the analysis, where relative risk aversion, risk tolerance, and the inverse of prudence play key roles in our results.

Keywords Changes in risk · Entrepreneurship · Stochastic dominance shifts · HARA preferences

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Introduction

The theory of entrepreneurship contains an ample spectrum of possible research questions to be tackled from the economic perspective, for example: the economic reasons why a worker may decide to become an entrepreneur, the way an

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entrepreneur may finance a new venture, the effects of entrepreneurship on economic development, or the importance of legal and economic institutions in the entrepreneurial process. For the most part, these economic analyses are based on the concepts of uncertainty, financial constraints, and the risk characteristics and features associated with the preferences of entrepreneurs and workers. The simple reason for this is that uncertainty and risk are key elements in the development of new ventures.

Knight (1921) was the first to develop the intuition about the connection between entrepreneurship and risk attitudes. Knight's idea was subsequently translated into economic models by Kanbur (1979) and Kihlstrom and Laffont (1979), and since then it has been a core component of the economic theory of entrepreneurship (Vereshchagina and Hopenhayn 2009; Koudstaal et al. 2015; Bonilla and Vergara 2021). The key intuition behind this theory is that the wealthy are, on average, less risk averse than the poor because well-behaved utility functions present the property of decreasing absolute risk aversion (DARA) and, therefore, the wealthy are more likely to launch new ventures.[1] Consequently, we should expect to observe mostly wealthy people choosing the occupation of entrepreneurship. The poor, in contrast, are more likely to become employees for a certain, fixed wage.

Most of the recent literature that uses microeconomic models of entrepreneurship based on risk attitudes builds on the DARA assumption and risk averse preferences. Following on from the original work of Kihlstrom and Laffont (1979), examples of the importance of these assumptions are the works of Cressy (2000), van Praag and Cramer (2001),Hartog et al. (2002), Kan and Tsai (2006), Ahn (2009), Caliendo et al. (2010), and Hvide and Panos (2014), among others. However, while this literature has focused on the effects of changes in model parameters (especially the entrepreneur's initial wealth and employee's given wage) and the shape of utility function, it has not focused on the effect of the shape of the underlying density function for entrepreneurial risk, which is something that may require assumptions beyond risk aversion and DARA to guarantee the expected comparative static results.

In this paper, we extend the entrepreneurship literature to consider the effects of changes in the risk underlying the problem of self-selection of occupations and entrepreneurship. The model used in this paper is based on Kihlstrom and Laffont (1979) and the stream of papers that follow but, instead of focusing on the comparative statics of the general parameters in the problem and the utility function, we concern ourselves with the changes in the shape of the underlying risk density under first- and second-order changes in the distribution of results.

Our results indicate that, under favorable risk changes (first- and second-order stochastic improvements) in the distribution of results, the expected comparative static results for both, the transition effect and the scale of the venture are not always satisfied with the assumptions of positive marginal utility, risk aversion, and DARA preferences. Therefore, we need additional conditions that involve levels of risk

[1] There is a second reason why the wealthy are more likely to become entrepreneurs, and that is because they face fewer financial constraints in securing capital (Evans and Jovanovic 1989). This line of research is outside the scope of this paper.

tolerance, relative risk aversion coefficients, and the inverse of the prudence index. We applied our general findings to hyperbolic absolute risk aversion (HARA) preferences and a specific distribution of results and clarified the intuition of our results.

The importance of studying the effects of risk shocks that change the distribution of results lies not only in theoretical reasons but also in the practical implications. For example, the COVID-19 pandemic shock affected sanitary conditions in all countries and it has provoked a previously unseen economic downturn worldwide. This shock can clearly be considered a negative shift in the distribution of results, and therefore, the study of the effects of changes in risk on entrepreneurship becomes of current importance to policymakers who want to speed up economic recovery through sound policy.

In the next section, we present the basic model of self-selection of occupation. Section 3 is dedicated to looking at first-order risk effects. There, we show what changes occur in the decision to become an entrepreneur/stay an employee (the transition effect) and in the optimal size of a transitioned entrepreneur's project (the scale effect) when the underlying density undergoes a first-order stochastic dominance (FOSD) change. In Sect. 4, we consider the comparative static of second-order risk effects of two types: a pure decrease in risk (a mean-preserving contraction, MPC), and a second-order stochastic dominance (SOSD) change in the density. Finally, Sect. 5 concludes.

The basic model of self-selection of occupations

Following the existing literature, we study an economy which is characterized by a single-good stochastic production function $f(L, \theta)$, where L is the labor hired and θ is a random variable indexing the state of the world and representing uncertainty in the model. We can think of θ as a random productivity shock that positively affects the production function and its corresponding marginal production: that is, $f(L, \theta)$ and its partial derivative $f_L(L, \theta)$ are increasing in the random productivity shock θ, and the production function satisfies $f_{LL} < 0 < f_L$. Given the assumptions of the model, an interior solution to the problem is expected. We assume that the price at which the good in question is sold is equal to 1, so that $f(L, \theta)$ also doubles as the revenue function.[2]. Since the only random variable in all of the analysis is θ, all expectations are taken with respect to that variable.

Agents are assumed to have identical preferences, represented by the utility function u. The utility function is assumed to satisfy $u'' < 0 < u'$ and the prudence property ($u''' > 0$) coined by Kimball (1990). This assumption is widely used in models of precautionary savings and precautionary effort, such as those by Eeckhoudt and Huang (2012) or Wang and Li (2014), and in models showing higher-order

[2] An alternative assumption, which we will exploit below, is that θ measures the price of the good: that is, revenue is $f(L, \theta) = \theta h(L)$, with $h(L)$ being an increasing concave production function. This example takes us to the classic literature of the firm under price uncertainty that started with Sandmo (1971)

risk attitudes, like in Menegatti (2014) or Eeckhoudt et al. (2016). We assume at all times that the decision maker maximizes expected utility.

The decision maker must choose between two occupations, either being an employee or becoming an entrepreneur (employer). Being an employee returns a certain income equal to the wage w, while an entrepreneur earns the residual profit from hiring L units of labor and selling the output resulting from a stochastic production function. Thus, the entrepreneur's final wealth is the money profit earned from entrepreneurship (the "payoff" function), which we denote by $\pi(L, w, \theta) = f(L, \theta) - wL$, where the stochastic component is the random variable θ.

There are, consequently, two main decisions to analyze: first, the decision of which occupational choice to make – employment or entrepreneurship. We denominate this choice as the "transition" effect, since we are interested in how changes in the stochastic environment determine the transition from employment to entrepreneurship. Second, if the decision maker has already transitioned into operating as an entrepreneur, then the decision as to the optimal employment of labor within the entrepreneurial project must be taken. We refer to this as the "scale" effect, since we associate the scale of the business with the amount of labor, L, hired.

When studying the transition effect, we begin by assuming the existence of a wage level w, such that the decision maker is indifferent between the two possible occupations. Therefore, at w, we have

$$Eu(\pi(L^*(w), \theta)) = u(w) \tag{1}$$

where $L^*(w)$ is the optimal scale given w, that is,

$$L^*(w) = \underset{\{L\}}{Argmax}\{Eu(\pi(L, w, \theta))\}.$$

The assumptions of the concavity of the utility function and of $f(L, \theta)$ in L imply that L^* is obtained by the following first-order condition:

$$E[u'(f(L^*(w), \theta) - wL^*(w))(f_L(L^*(w), \theta) - w)] = 0 \tag{2}$$

Using essentially this same model, but explicitly including additional non-random initial wealth, Kihlstrom and Laffont (1979) proved a number of comparative static results for this problem. Specifically, the following all hold in the present model[3]:

1. An increase in risk aversion will lead to a lower propensity to transition into entrepreneurship and results in smaller projects for agents engaged in entrepreneurship.
2. An increase in the risk-free wealth of an agent will, assuming DARA, lead to a higher propensity to transition into entrepreneurship and results in larger projects for agents engaged in entrepreneurship.

[3] Proofs for all of these results—in a partial equilibrium analysis—can be found in Bonilla et al. (2020). Some of these results have been found by other authors over the years in a variety of similar theoretical models (see, for example, Cressy (2000), and Newman (2007)). Others have tested the validity of these results empirically (see, for example, van Praag et al. (2002), and Kan and Tsai (2006)).

3. An increase in the salary paid to employees will lead to a lower propensity to transition into entrepreneurship, and it will reduce the optimal size of entrepreneurial ventures if the utility function of entrepreneurs is DARA, or if the utility function satisfies relative risk aversion no larger than 1.

Throughout the existing literature, the focus has been on studying the transition and scale effects under changes in the parameters of the problem, such as initial wealth, the equilibrium wage and the form of the utility function (i.e., the effect of greater risk aversion). In those analyses, the underlying risk structure pertaining to the entrepreneur's distribution of results has typically been held constant. However, it is intuitive that a change in the distribution of risk will affect both the optimal scale of the venture and the decision of whether to transition into entrepreneurship. To undertake that study, we will assume two different kinds of shift in the underlying density corresponding to the choice of being an entrepreneur: first-order shifts (FOSD shifts) and second-order shifts (either pure decreases in risk or SOSD shifts). Since in this paper we are only interested in shifts of the results' distribution function, we hold the wage parameter constant, dropping it from our equations entirely. Therefore, unless we want to specify concrete cases where w needs to be made explicit, we will simply use $\pi(L,\theta)$ as the variable indicating the (random) final entrepreneurial income or residual profits of entrepreneurship, $\pi(L,\theta) = f(L,\theta) - wL$.

In most of what follows, we will assume that our random variable has the structure $\theta = \bar{\theta} + \gamma\epsilon$, where ϵ is a random variable with $E(\epsilon) = 0$. This linear decomposition of the random variable θ has two advantages. First, it connects our paper with a large stream of classic literature on decision making under risk.[4] And second, it provides us with a clear intuition of the difference between a FOSD improvement (an increase in $\bar{\theta}$) and a MPC[5] (a reduction in γ).

To facilitate the intuition of our analysis when possible, we will discuss our results in the context of comparing two different densities for θ. We will capture this by labeling the random variable with a subscript indicating its underlying density, θ_i for $i = 1, 2$. The initial situation will be $i = 1$, and the comparator will be $i = 2$. For example, when we want to consider a pure decrease in risk (a MPC), we will use θ_2 to be a MPC of θ_1. In general, for each comparison, we can identify the effect of moving from a given density to either a more or a less preferred one. The default comparison will be moving from an initial risk distribution to another which is, in principle, more favorable. Accordingly, for example, we will look at θ_2 dominating

[4] This parametric class of distribution has been used in the literature since Sandmo (1971) introduced such linear decomposition in the theory of the firm under price uncertainty. More recent uses of this formulation can be seen in Baiardi et al. (2020). Nonlinear risks and higher-order risk preferences are examples of more complex risk structures. Using Sandmo's risk structure, on the other hand, provides a simpler and clearer intuition of the results obtained

[5] An alternative way to model FOSD changes and MPC is the two-moment (or the mean-variance) decision model. Since Meyer (1987) we have known that, under certain conditions, we can transit from the traditional expected utility model to the two-moment decision model to analyze optimal decisions under risk. See for example Broll et al. (2015) and Vergara and Bonilla (2021) for applications to the banking firm and precautionary saving, respectively

θ_1 according to the stochastic dominance criteria (first- or second-order), or θ_2 being a reduction in pure risk compared to θ_1 – a mean-preserving contraction. Of course, any result obtained for moving to a better density has a parallel result for moving in the opposite direction.

First-order effects

The initial situation is one in which the underlying risk in the entrepreneurial activity is given by θ_1. Given that risk, we want to study the effect of moving to θ_2, which dominates θ_1 in the sense of first-order stochastic dominance.

The transition effect

Intuitively, a positive stochastic dominance shift in the density will lead to a greater propensity to switch from employment to entrepreneurship, and a negative stochastic dominance shift will lead to the opposite. We show now that, for FOSD, this is indeed so.

Proposition 1 *A first-order stochastic dominance shift in the distribution of θ induces the indifferent decision maker to transition into risky entrepreneurship.*

Proof Assume that the decision maker is initially indifferent between employment and entrepreneurship:

$$Eu(\pi(L_1^*, \theta_1)) = u(w)$$

Now, our assumptions on utility are that it is strictly increasing and strictly concave in profits, $\pi(L, \theta)$. Furthermore, profits are increasing in θ, since $\pi_\theta(L, \theta) = f_\theta(L, \theta) > 0$. Thus we know that utility is increasing in θ, $u_\theta(\pi(L, \theta)) = u'(\pi(L, \theta))\pi_\theta(L, \theta) > 0$. If θ switches from θ_1 to θ_2 (with $\bar{\theta}_1 < \bar{\theta}_2$), we state that θ_2 dominates θ_1 under first-order stochastic dominance. Then, because utility is increasing in θ, it must occur that

$$Eu(\pi(L_1^*, \theta_2)) > Eu(\pi(L_1^*, \theta_1))$$

If we now allow the individual to choose the optimal scale of the firm under θ_2, we get

$$Eu(\pi(L_2^*, \theta_2)) > Eu(\pi(L_1^*, \theta_2))$$

Finally, substituting back into the originally indifferent individual, we now know that

$$Eu(\pi(L_2^*, \theta_2)) > Eu(\pi(L_1^*, \theta_1)) = u(w)$$

In other words, a first-order stochastic dominance improvement in density causes previously indifferent individuals to strictly prefer to become entrepreneurs. □

Of course, this simultaneously proves the converse statement: if θ undergoes a negative first-order stochastic dominated shift, then the initially indifferent decision maker is induced into employment rather than entrepreneurship.

The scale effect

Now assume that an individual has already transitioned into entrepreneurship, and we ask how a first-order stochastic dominance shift in the density affects the optimal size of the business venture (as measured by the employment of labor). It is very tempting to initially assume that a first-order stochastic dominance improvement in density will lead to a larger business scale. Here we show that this might not be the case; or, at least, that to guarantee it, we need to condition the functions involved. The reason for this difference with the previous section is that the transition effect requires us only to keep track of how the changes affect the value of utility, but the scale effect requires us to understand how the changes affect marginal utility.

To study this effect, we need to remember that the structure of our random variable is $\theta = \bar{\theta} + \gamma\epsilon$, where a small FOSD shift in the distribution of θ has an effect on firm profits, given by $\pi_{\bar{\theta}} = \pi_\theta$. We also need to consider the first-order condition for the optimal scale:

$$Eu'(\pi(L^*, \theta))\pi_L(L^*, \theta) = 0$$

We can now study the effects of stochastic dominance shifts by considering how such a change affects the value of the function $Eu'(\pi(L, \theta))\pi_L(L, \theta)$.

From the second-order condition for an optimal choice of labor, we know that $Eu'(\pi(L, \theta))\pi_L(L, \theta)$ decreases with L. If, additionally, we assume (for now) that $u'(\pi(L, \theta))\pi_L(L, \theta)$ increases with $\bar{\theta}$, then for any given L, a FOSD shift in density must increase $Eu'(\pi(L, \theta))\pi_L(L, \theta)$. Therefore, a FOSD shift in density will cause the expected value of $Eu'(\pi(L^*, \theta))\pi_L(L^*, \theta)$ to become strictly positive rather than 0, as would be required for an optimal scale. The entrepreneur will adjust L to bring the expected value of $Eu'(\pi(L^*, \theta))\pi_L(L^*, \theta)$ back to 0, and since $Eu'(\pi(L^*, \theta))\pi_L(L^*, \theta)$ decreases with L, this is achieved by increasing L^*.

Therefore, in order for a FOSD shift to increase the optimal scale of the enterprise, it is sufficient that $u'(\pi(L, \theta))\pi_L(L, \theta)$ is increasing in $\bar{\theta}$. This leads us to the following proposition:

Proposition 2 *A first-order stochastic dominance shift in the density of θ (i.e., an increase in $\bar{\theta}$) will increase the optimal scale of the venture, subject to the condition that* **absolute risk tolerance is sufficiently high**.

Proof The derivative of $u'(\pi(L^*, \theta))\pi_L(L^*, \theta)$ with respect to $\bar{\theta}$ is

$$u''(\pi(L^*, \theta))\pi_\theta(L^*, \theta)\pi_L(L^*, \theta) + u'(\pi(L^*, \theta))\pi_{L\theta}(L^*, \theta) \tag{3}$$

Since $\pi_\theta(L^*, \theta) = f_\theta(L^*, \theta)$, $\pi_L(L^*, \theta) = f_L(L^*, \theta) - w$, and $\pi_{L\theta}(L^*, \theta) = f_{L\theta}(L^*, \theta)$, condition (3) becomes

$$u''(\pi(L^*,\theta))f_\theta(L^*,\theta)\big(f_L(L^*,\theta)-w\big)+u'(\pi(L^*,\theta))f_{L\theta}(L^*,\theta) \quad (4)$$

In order to prove Proposition 2, we require equation (4) to be positive. We need to re-order this equation but, in doing so, notice that it would be unwise to multiply or divide by $f_L(L^*,\theta)-w$, since that term has an ambiguous sign and would therefore affect the inequality direction in a different way, depending on its sign. However, moving the utility terms to one side and the terms involving the derivatives of the production function to the other, we find that the condition is

$$F(\theta)\equiv\frac{f_\theta(L^*,\theta)\big(f_L(L^*,\theta)-w\big)}{f_{L\theta}(L^*,\theta)}<-\frac{u'(\pi(L^*,\theta))}{u''(\pi(L^*,\theta))}=T(\pi(L^*,\theta)) \quad (5)$$

where $T(\pi)$ is absolute risk tolerance.[6] □

Notice, first of all, that the condition required for a FOSD improvement to increase the size of the venture, equation (5), is in fact a relationship between functions (of θ), not simply a condition of function values. That is, the condition should be satisfied for all values of θ if the proposition is to hold true. Second, our assumption throughout is that $f_{L\theta}>0$, so at small values of $\bar{\theta}$ we get $f_L(L^*,\theta)-w<0$. The condition will be satisfied trivially so long as risk tolerance is positive, which is guaranteed by virtue of the assumption that the utility function is increasing and concave. The interesting cases, then, are where $\bar{\theta}$ takes larger values, and it is unclear if the condition will be satisfied in general because both sides of the inequality will be positive.

Application to a HARA function and a random price

To illustrate this last proposition, assume that utility is of the (commonly assumed) HARA class, so that it has linear risk tolerance: $T(\pi(\theta))=m+b\pi(\theta)=m+b(f(L,\theta)-wL)$, where m and b are positive constants. Then assume that the risk in question is in fact a risky price, so that $f(L,\theta)=\theta h(L)$, with h increasing and concave and $\theta=\bar{\theta}+\gamma\epsilon$. Given these assumptions, we have $f_L=\theta h'(L)$, $f_\theta=h(L)$, $f_{L\theta}=h'(L)$, and $T_\theta(\pi(\theta))=bh(L)$.

[6] We could also prove this proposition by using Diamond and Stiglitz (1974)'s definitions. Let us define a change in τ as a first-order risk improvement if $F_\tau(\theta,\tau)<0$ and $F_\tau(a,\tau)=F_\tau(b,\tau)=0$. Let us also define $U(\theta,L)=u(\pi(\theta,L))$. Then, applying the implicit function theorem to the first-order condition, we get:

$$\frac{dL^*}{d\tau}=-\frac{\int_a^b U_L dF_\tau(\theta,\tau)}{SOC}$$

where SOC stands for the second-order condition of the problem, which is negative. Then, $sign(\frac{dL^*}{d\tau})=sign(\int_a^b U_L dF_\tau(\theta,\tau))$. Integrating the expression $\int_a^b U_L dF_\tau(\theta,\tau)$ by parts, we get $-\int_a^b U_{L\theta}F_\tau d\theta$, which is positive as long as $U_{L\theta}=u''(\pi)\pi_\theta\pi_L+u'(\pi)\pi_{\theta L}>0$, and working on this expression, we get the same condition as (5) above.

Then, the condition required for a FOSD improvement in θ (increase in $\bar{\theta}$) to increase the size of the business, i.e., equation (3), becomes

$$\frac{h(L)\big(\theta h'(L) - w\big)}{h'(L)} < m + b(\theta h(L) - wL) \tag{6}$$

The derivative of the left-hand side of (6) in θ is just $h(L)$, and the derivative of the right-hand side is $bh(L)$. Now, tolerance is everywhere positive, while the left-hand side expression is negative at a small θ , so clearly the condition is satisfied for a small θ. Indeed, if $b \geq 1$, tolerance grows faster than the left-hand side, so the condition is always satisfied. But if $b < 1$, then tolerance grows more slowly than the left-hand side, and at a certain value of θ, the condition will no longer be satisfied.

The problem of $b < 1$ can be addressed by noticing that there is a relationship between this example and relative risk aversion. Concretely, under the assumptions of HARA utility and $f(L, \theta) = \theta h(L)$, the condition for a FOSD improvement to increase the size of the business, equation (6), is

$$\frac{h(L)\big(\theta h'(L) - w\big)}{h'(L)} < T(\pi(\theta))$$

which can be written as

$$\theta h(L) - \frac{h(L)}{h'(L)} w < T(\pi(\theta)). \tag{7}$$

But, since $h(L)$ is concave, $Lh'(L) < h(L)$, that is, $\frac{h(L)}{h'(L)} > L$. Therefore,

$$\theta h(L) - \frac{h(L)}{h'(L)} w < \theta h(L) - Lw$$

and the above condition (7) can be guaranteed if

$$\theta h(L) - Lw < T(\pi(\theta)) = \frac{u'(\pi(\theta))}{-u''(\pi(\theta))}$$

We can now cross-multiply so that this reads

$$\frac{-u''(\pi(\theta))}{u'(\pi(\theta))}(\theta h(L) - Lw) < 1$$

Finally, since $\theta h(L) - Lw = \pi(\theta)$, we have

$$\frac{-u''(\pi(\theta))}{u'(\pi(\theta))}(\theta h(L) - Lw) = R(\pi(\theta)) < 1$$

where, of course, $R(\pi(\theta))$ is the Arrow-Pratt measure of relative risk aversion. This leads us directly to the following:

Proposition 3 *If (i) relative risk aversion is no greater than 1, (ii) utility is within the HARA class, and (iii)* $f(L,\theta)=\theta h(L)$ *with h increasing and concave, then a first-order stochastic dominance improvement of the density of* θ *will lead to a larger size of the entrepreneur's venture.*

Again, naturally, the converse result also applies: under the same conditions of linear risk tolerance and with income being linear in risk, a first-order dominated shift will reduce the size of the business.

Second-order effects

We now move on to second-order changes in risk. The economics literature in general has considered two categories of second-order risk effects: pure changes in risk (increases or decreases in risk that leave the expected value constant), which are either a mean-preserving spread (MPS) or a mean-preserving contraction (MPC); and second-order stochastic dominance, in which both risk and expected value may change. Therefore, a pure change in risk is just a special case of SOSD. We can also note that any general SOSD effect can always be decomposed into a pure change in risk and a FOSD shift (see Shaked and Shanthikumar (2007), Theorem 4.A.6. See also Liu and Meyer (2015), Theorem 2). Given that decomposition result, if a pure reduction in risk serves to enhance the transition into entrepreneurship, then so will a general SOSD shift (from θ_1 to θ_2, where θ_2 second-order stochastic dominance θ_1), since we have already shown that a FOSD improvement will have that effect. On the other hand, if a MPC serves to increase the optimal scale of the business, then, under the same condition as in Proposition 2 above, so will a general SOSD improvement increase the scale of the business. With that in mind, we now undertake an analysis of the effects of a MPC.

Following Rothschild and Stiglitz (1970), a pure decrease in risk is defined by a MPC in the density of the random variable θ. Let θ_i be the random variable under density i, where $i = 1, 2$, and where density 2 is a MPC of density 1. Let L_i^* be the optimal choice of scale of the entrepreneurial project under density i.

Transition effect

Assume, as in the previous section, that the decision maker is initially indifferent between the options of entrepreneurship and employment:

$$Eu\big(\pi(L_1^*,\theta_1)\big)=u(w)$$

Since θ_2 is a mean-preserving contraction of θ_1, it happens that, for any decision maker, θ_2 is preferred to θ_1 if utility is concave in θ (Rothschild and Stiglitz 1970). Since L_i^* is the optimal choice under random variable θ_i, then $Eu\big(\pi(\theta_2,L_2^*)\big)\geq Eu\big(\pi(\theta_2,L_1^*)\big)$. Now, *assuming* (for now) that utility is concave in θ, we also have $Eu\big(\pi(\theta_2,L_1^*)\big)>Eu\big(\pi(\theta_1,L_1^*)\big)$. Therefore, under the concavity assumption of θ, we end up with $Eu\big(\pi(\theta_2,L_2^*)\big)>Eu\big(\pi(\theta_1,L_1^*)\big)$, which means that the switch to a density with less pure risk must increase the expected utility of a risk-averse entrepreneur. Since the decision maker was initially indifferent between entrepreneurship and employment, the reduction in pure risk in the entrepreneurship option makes him strictly prefer entrepreneurship over employment. Of course, the reverse argument also holds: an increase in pure risk in the entrepreneurship option leads to a greater preference for employment over entrepreneurship for an initially indifferent decision maker, assuming that utility is concave in θ.

In the parametric class case, this argument depends critically upon the assumption that utility $u(\pi(L,\theta))$ is concave in γ. Since $\theta = \bar{\theta} + \gamma\epsilon$, the effect of a small MPC in the distribution of θ (in other words, a small decrease in γ) has an effect on firm's profits given by $\pi_\gamma = \pi_\theta\epsilon$. The first derivative of the utility function in γ is given by $u'(\pi)\pi_\theta(L,\theta)\epsilon$.[7] The second derivative is $u''(\pi)\pi_\theta(L,\theta)^2\epsilon^2 + u'(\pi)\pi_{\theta\theta}(L,\theta)\epsilon^2$. This is what we require to be negative for the concavity of utility in γ. Clearly, the first term is negative from the assumption of risk aversion. Then, a decrease in pure risk increasing the utility of entrepreneurship will always hold if $\pi_{\theta\theta}(L,\theta) \leq 0$ (sufficient condition). In our problem, $\pi_{\theta\theta}(L,\theta) = f_{\theta\theta}$, so what is required is for the production/revenue function be not (strictly) convex in θ, i.e., it is either concave or linear. Naturally, a parallel argument applies for a pure increase in risk (a MPS).

In short, we have:

Proposition 4 *A pure decrease in the risk of the distribution of entrepreneurial outcomes leads to a greater propensity for transition into entrepreneurship if $f(L,\theta)$ is not (strictly) convex in γ.*

Given that, as outlined above, any SOSD effect can be decomposed into a FOSD effect together with a change in pure risk, we also immediately arrive at the following:

Proposition 5 *A SOSD improvement in the distribution risk of entrepreneurial outcomes leads to a greater propensity for transition into entrepreneurship if $f(L,\theta)$ is not (strictly) convex in θ.*

The expected parallel result holds for SOSD worsening.

An obvious special case emerges when θ is a random price level, so $f(L,\theta) = \theta h(L)$ is a revenue function with production function $h(L)$. In this case, $f(L,\theta)$ is linear in θ, $f_{\theta\theta} = 0$, and the sufficient condition for a second-order dominant effect to increase the propensity for transition into entrepreneurship holds directly.

Of course, non-convexity of $f(L,\theta)$ in θ is only a sufficient condition for the logical second-order effect on transition to work. It is also useful to propose an alternative sufficient condition that is based on risk aversion, since, clearly, the more risk averse the decision maker is, the greater the impact of a second-order shift

[7] The derivative of the expected utility function in γ is given by $EU_\theta\epsilon = cov(U_\theta,\epsilon)$, where $U(\theta,L) = u(\pi(\theta,L))$. Thus, $cov(U_\theta,\epsilon) > (<)0$, if and only if, $U_{\theta\theta}\gamma = u''(\pi)\pi_\theta^2\gamma + u'(\pi)\pi_{\theta\theta}\gamma > (<)0$.

on the risk in question. To do this, recall that a MPC will enhance the transition effect if $u''(\pi)\pi_\theta(L,\theta)^2\epsilon^2 + u'(\pi)\pi_{\theta\theta}(L,\theta)\epsilon^2 < 0$. We know that $u''(\pi) < 0$, $\epsilon^2 > 0$, $\pi_\theta(L,\theta)^2 > 0$,and $u'(\pi) > 0$, but we cannot be sure of the sign of $\pi_{\theta\theta}(L,\theta)$. Therefore, it is unwise to multiply or divide by $\pi_{\theta\theta}(L,\theta)$. However, we can still re-order the required condition to read

$$\frac{\pi_{\theta\theta}(L,\theta)}{\pi_\theta(L,\theta)^2} < -\frac{u''(\pi)}{u'(\pi)} = A(\pi)$$

In other words, absolute risk aversion needs to be sufficiently high (and if $\pi_{\theta\theta}(L,\theta) \le 0$, the condition holds with positive risk aversion). However, this condition becomes more meaningful if we first multiply both sides by π and then subtract 1 from each side:

$$\frac{\pi_{\theta\theta}(L,\theta)\pi}{\pi_\theta(L,\theta)^2} - 1 < -\frac{u''(\pi)\pi}{u'(\pi)} - 1 = R(\pi) - 1$$

where –again– $R(\pi)$ is relative risk aversion.

Now, if relative risk aversion is not less than 1, $R(\pi) \ge 1$, then it is sufficient for the desired result that[8]

$$\frac{\pi_{\theta\theta}(L,\theta)\pi}{\pi_\theta(L,\theta)^2} < 1$$

This leads to the following alternative sufficient condition:[9]

Proposition 6 *A pure decrease in the risk of the entrepreneurial option leads to a greater propensity for transition into entrepreneurship if relative risk aversion is not less than 1* and $(\frac{\pi_\theta}{\pi})$ *is non-increasing in* θ.

Proof All that is required is to derive $\left(\frac{\pi_\theta}{\pi}\right)$ with respect to θ:

$$\frac{d}{d\theta}\left(\frac{\pi_\theta}{\pi}\right) = \frac{\pi_{\theta\theta}\pi - \left(\pi_\theta\right)^2}{\pi^2}$$

This is negative if

$$\pi_{\theta\theta}\pi < \left(\pi_\theta\right)^2$$

or

$$\frac{\pi_{\theta\theta}\pi}{\pi_\theta^2} < 1$$

□

8 Empirical estimates of the relative risk aversion coefficient vary depending on the period, place, and source of the data used. However, most estimates range between 1 and 3. For instance Chiappori and Paiella (2011), using data from the Bank of Italian Survey of Household Income and Wealth, estimate the RRA in a range between 2 and 3. Conine et al. (2017). estimate RRA using a long time series for the US with an estimate of between 2 and 2.23.

9 We thank Richard Peter, who suggested this way of looking at the sufficient condition.

This sufficient condition can be described as follows: a MPC in θ will enhance the transition into entrepreneurship if relative risk aversion is no less than 1 and if the rate at which profit grows with θ (which is $\frac{\pi_\theta}{\pi}$) is decreasing in θ.

Of course, a companion proposition also holds for a general SOSD change in the density:

Proposition 7 *A SOSD improvement in the risk of the entrepreneurial option leads to a greater propensity for transition into entrepreneurship if relative risk aversion is not less than 1 and* $\left(\frac{\pi_\theta}{\pi}\right)$ *is non-increasing in* θ.

The scale effect

It is equally straightforward to consider the effect upon the optimal scale of a transitioned entrepreneur's venture if there is a mean-preserving contraction in the density of θ, that is, the question of which is greater between L_1^* and L_2^*. However, the outcome is now ambiguous and depends on higher-order derivatives of the decision problem. Specifically, we only need to directly apply the result of Rothschild and Stiglitz (1971), which in the current context is the following:

Proposition 8 *A pure decrease in the risk of the distribution of entrepreneurial outcomes leads to a larger optimal scale if the marginal utility of the decision maker,* $u'(\pi(L,\theta))\pi_L(L,\theta)$*, is concave in* γ.

The logical, or more intuitive effect of a MPC is that the optimal scale should increase (and, conversely, a MPS should decrease the optimal scale). From the above proposition, this will only be the case if $u'(\pi(L,\theta))\pi_L(L,\theta)$ is concave in γ, that is, if the second derivative of $u'(\pi(L,\theta))\pi_L(L,\theta)$ with respect to γ is negative. It is simple to see that the implied second derivative will contain the second and third derivatives of u, and the first, second, and third derivatives of $f(L,\theta)$ (some being cross-derivatives). In short, it will be a collection of terms, some of which are positive and others are negative. Also, while we have made assumptions on the second and third derivatives of utility (respectively, negative and positive), and we have made some assumptions on the slopes of $f(L,\theta)$, we would clearly have to make further assumptions if we are to arrive at a determination for concavity or convexity of the marginal utility scale in θ.

However, one relatively simple (and potentially rather important and interesting) special case is easy to spot. Assume that $\theta = \bar{\theta} + \gamma\epsilon$ is a random price of the entrepreneur's output, so that $f(L,\theta) = \theta h(L)$ is the revenue function and $h(L)$ is the (increasing and concave) production function. Also, keep in mind that $\pi_\gamma = \pi_\theta\epsilon$. In that case, we get the following alternative proposition[10]:

[10] The derivative of $Eu'(\pi)\pi_L$ in γ is given by $EU_{L\theta}\epsilon = cov(U_{L\theta},\epsilon)$, where $U_L(\theta,L) = u'(\pi)\pi_L$. Thus, $cov(U_{L\theta},\epsilon) > (<)0$ if and only if $U_{L\theta\theta}\gamma > (<)0$, where $U_{L\theta\theta} = u'''(\pi)\pi_\theta^2\pi_L + u''(\pi)\pi_{\theta\theta}\pi_L + 2u''(\pi)\pi_\theta\pi_{L\theta} + u'(\pi)\pi_{L\theta\theta}$.

Proposition 9 *If $f(L,\theta)=\theta h(L)$, then a MPC in the entrepreneurial distribution of results leads to a larger optimal scale if*

$$\frac{(\theta h'(L)-w)h(L)}{2h'(L)} < -\frac{u''(\pi)}{u'''(\pi)}$$

Proof The second derivative of $u'(\pi(L,\theta))\pi_L(L,\theta)$ with respect to γ is:

$$u'''(\pi)\pi_L(L,\theta)\pi_\theta(L,\theta)^2\epsilon^2+u''(\pi)\pi_{\theta\theta}(\theta,L)\pi_L(L,\theta)\epsilon^2+u''(\pi)\pi_\theta(L,\theta)\pi_{L\theta}(L,\theta)\epsilon^2 \\ +u''(\pi)\pi_\theta(L,\theta)\pi_{L\theta}(L,\theta)\epsilon^2+u'(\pi)\pi_{L\theta\theta}(\theta,L)\epsilon^2 \tag{8}$$

and in this case, with $f(L,\theta)=\theta h(L)$, so that $\pi(\theta,L)=\theta h(L)-wL$, then

$$\pi_L=\theta h'(L)-w;\ \pi_{L\theta}=h'(L);\ \pi_\theta=h(L);\ \text{and}\ \pi_{\theta\theta}=\pi_{L\theta\theta}=0$$

Therefore, the previous expression (8) simplifies to

$$u'''(\pi)\big(\theta h'(L)-w\big)h(L)^2+2u''(\pi)h'(L)h(L)$$

and for concavity, we require this to be negative, which, after some simplification, yields

$$\frac{(\theta h'(L)-w)h(L)}{2h'(L)} < -\frac{u''(\pi)}{u'''(\pi)}$$

– as required.[11] □

Thus, what is of the essence for this special case is that the inverse of absolute prudence should be sufficiently large. We will revisit this condition shortly.

Notice that the condition required in Proposition 9 is just

$$\int_a^b F_\tau(\theta,\tau)d\theta=0$$

and

$$T(\theta,\tau)=\int_a^\theta F_\tau(z,\tau)dz\le 0 \quad \text{for all} \quad a\le\theta\le b$$

and also, $T(a,\tau)=T(b,\tau)=0$. The first condition indicates that both distributions exhibit the same mean, while the second is the single-crossing property of a mean-preserving contraction. Let us define $U(\theta,L)=u(\pi(\theta,L))$. Then, applying the implicit function theorem to the first-order condition, we get

$$\frac{dL^*}{d\tau}=-\frac{\int_a^b U_L dF_\tau(\theta,\tau)}{SOC}$$

Applying integration by parts twice to the numerator, and given that $F_\tau(a,\tau)=F_\tau(b,\tau)=T(a,\tau)=T(b,\tau)=0$, we obtain $\int_a^b U_{L\theta\theta}T_{\theta,\tau}d\theta$, which is positive as long as $U_{L\theta\theta}<0$, where $U_{L\theta\theta}=u'''(\pi)\pi_\theta^2\pi_L+u''(\pi)\pi_{\theta\theta}\pi_L+2u''(\pi)\pi_\theta\pi_{L\theta}+u'(\pi)\pi_{L\theta\theta}$, which is exactly Eq. (8).

[11] An alternative proof of the general case can be done using Diamond and Stiglitz (1974), where the change in τ is considered a second-order risk improvement (or a mean-preserving contraction) if

$$\frac{F(\theta)}{2} < -\frac{u''(\pi(\theta))}{u'''(\pi(\theta))}$$

where $F(\theta)$ is the function constraining risk tolerance in Proposition 2 above:

$$F(\theta) = \frac{f_\theta(L,\theta)(f_L(L,\theta) - w)}{f_{L\theta}(L,\theta)}$$

To make sense of this, it would be useful to understand what the inverse of prudence is, and how it relates to risk tolerance. To that end, we have the following lemma.

Lemma 1 *For any utility function, the measure of absolute prudence, $P(\pi)$, can be expressed as*

$$P(\pi) = \frac{T'(\pi) + 1}{T(\pi)}$$

where $T(\pi)$ is absolute risk tolerance.

Proof Notice that

$$T'(\pi) = \frac{d}{d\pi}\left(-\frac{u'(\pi)}{u''(\pi)}\right) = -\left(\frac{u''(\pi)u''(\pi) - u'(\pi)u'''(\pi)}{u''(\pi)^2}\right)$$

Consequently, the slope of tolerance is

$$\begin{aligned} T'(\pi) &= -\left(1 - \frac{u'(\pi)}{u''(\pi)}\frac{u'''(\pi)}{u''(\pi)}\right) \\ &= \frac{u'(\pi)}{u''(\pi)}\frac{u'''(\pi)}{u''(\pi)} - 1 \\ &= \left(-\frac{u'(\pi)}{u''(\pi)}\right)\left(-\frac{u'''(\pi)}{u''(\pi)}\right) - 1 \\ &= T(\pi)P(\pi) - 1 \end{aligned}$$

where $P(\pi)$ is absolute prudence. Therefore, we get

$$P(\pi) = \frac{T'(\pi) + 1}{T(\pi)}$$

□

Notice that, from Lemma 1, we know that the inverse of prudence is directly related to risk tolerance by

$$\frac{1}{P(\pi)} = \frac{T(\pi)}{T'(\pi) + 1}$$

And since, under the HARA class of utility functions, tolerance is linear, then $T'(\pi) = b$, where b is a constant. So, with HARA preferences, the inverse of prudence is a linear function of tolerance, and it is everywhere less than tolerance (assuming, of course DARA, or $T'(\pi) > 0$). Given this, we can now state the next lemma, which follows directly from our previous results:

Lemma 2 *Assuming HARA preferences, with the slope of risk tolerance equal to a constant b, and if the production function is linear in θ, then a second-order stochastic dominance shift in the density of θ will increase the optimal scale of the venture if*

$$F(\theta) < \frac{2}{b+1} T(\pi(\theta))$$

If, for example, $b = 1$, then we find that exactly the same condition guarantees that either a first- or a second-order stochastic dominance shift results in an increase in the size of the business. More specifically, using the same argument as in Proposition 9 above, we can establish a sufficient condition concerning the effect of SOSD and relative risk aversion:

Proposition 10 *Assuming HARA utility, with the slope of risk tolerance equal to a constant b and assuming that the production function is linear in θ, then a second-order stochastic dominant shift in the density of θ will increase the optimal scale of the business if relative risk aversion is not greater than* $\left(\frac{2}{b+1}\right)$.

Proof From Lemma 2 above, write the sufficient condition for a second-order stochastic dominance shift to increase the scale of the business as

$$\frac{h(L)(\theta h'(L) - w)}{h'(L)} < \frac{2}{b+1} T(\pi(\theta)) \quad \Longrightarrow \quad h(L)\theta - \frac{h(L)w}{h'(L)} < \frac{2}{b+1} T(\pi(\theta))$$

Given that the production function, $h(L)$ is concave, then $\frac{h(L)}{h'(L)} > L$, so $h(L)\theta - \frac{h(L)w}{h'(L)} < h(L)\theta - Lw = \pi(\theta)$. Therefore, our condition is guaranteed if $\pi(\theta) \leq \frac{2}{b+1} T(\pi(\theta))$, which we express as

$$\frac{\pi(\theta)}{T(\pi(\theta))} \leq \frac{2}{b+1} \text{ i.e. } R(\pi(\theta)) \leq \frac{2}{b+1}$$

where –again– $R(\pi)$ is the relative risk aversion coefficient. □

If, for example, $b \leq 1$, then we would have $\left(\frac{2}{b+1}\right) \geq 1$, and in this case both of the sufficient conditions (for FOSD and SOSD) for the optimal scale to increase under a stochastic dominance shift in the density are satisfied whenever relative risk aversion is no greater than 1.

Concluding remarks

We have revisited the literature on the choices of economic agents regarding entrepreneurship. Specifically, we extend the literature to consider how stochastic dominance shifts in the underlying entrepreneurial risk affect both transition and scale. We found that transition follows the expected intuition: a stochastic dominance shift of either first or second order will increase the propensity to become an entrepreneur. However, the scale effect is much more interesting under stochastic dominance. There, we find that, in order to guarantee the intuitive outcome –namely, that a positive stochastic dominance shift increases the optimal scale of the venture–, it is necessary to condition risk tolerance to be sufficiently high. To illustrate that result, we have appealed to the case of HARA utility and risk that enters the decision-making process linearly. In that case, the FOSD effect can be guaranteed if relative risk aversion is no greater than 1. The analysis of a SOSD shift is far more complex, and we have limited our pertinent results to when utility is in the HARA class and risk enters the problem linearly. We find that the same threshold reappears for the condition on FOSD but this time in relation to the inverse of absolute prudence. We also show that, in the special case of HARA and linear risk, our sufficient conditions for stochastic dominance can be expressed in terms of relative risk aversion being not greater than 1.

Our analysis suggests plenty of scope for extentions. Mainly, it would be of interest to resolve the issue of SOSD generally, that is, without having to appeal to HARA preferences and linear risks. We could easily express the condition that is required (that the first derivative of utility with respect to labor should be concave in risk), but analyzing what that actually implies for the shapes of utility and the production/revenue function is more complex. Another possible extension is the incorporation of a two-moment utility model that study changes in the mean and the variance and their effects in the entrepreneur's optimal decision

Finally, another interesting extension of this work is to analyze the effect of higher-order risk improvements on entrepreneurship. For higher-order cases, the formulation of the stochastic variable $\theta = \bar{\theta} + \gamma\epsilon$ is not enough to capture any stochastic dominance shift of order $n \geq 3$ and, therefore, a different approach must be developed to tackle the problem. However, we consider that the two cases developed in this paper (FOSD and SOSD) are an important first step into a better understanding of how stochastic shocks affect entrepreneurial decision making and extend the current literature concerning risk and entrepreneurship.

Declarations

Conflict of interest On behalf of all authors, the corresponding author states that there is no conflict of interest

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